2009 NORTH AMERICAN ANNUAL MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

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Abstracts of invited plenary speakers

► PATRICIA BLANCHETTE, *Logical entailment and conceptual analysis in Frege.*
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Frege thought that the logical-entailment relation between propositions turned to some extent on relations of conceptual containment between parts of those propositions. Hence on this view the analysis of mathematical concepts plays an important role in answering questions of logical entailment in mathematics. This relatively-rich conception of entailment carries with it a non-standard view of various modern questions and results, including those of consistency, independence, and completeness. This talk aims to clarify some of these implications of the view, and to assess their virtues and vices.

► KIT FINE, *The logic of vagueness.*
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I discuss a new approach to the logic of vagueness.

► JAN KRAJÍČEK, *Forcing with random variables in bounded arithmetic, and proof complexity.*
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Bounded arithmetic and proof complexity (tacitly propositional) are very much interconnected. In particular, the task of proving lower bounds is for any particular proof system P equivalent to the task of constructing suitably non-elementary extensions of models of a bounded arithmetic T_P theory depending on P. Many lower bounds can be explained very naturally as constructions of such extensions and, in fact, quite a few were discovered in this way.

Models M to be extended are cuts in non-standard models of true arithmetic. The wanted extensions N of M should preserve polynomial-time properties but should not be elementary w.r.t. $\mathcal{NP}$-properties. If we target lower bounds for proof system P, models N should satisfy bounded arithmetic theory T_P. This makes the task difficult as theory T_P can be quite strong even for a simple proof system P.

I shall describe a method for constructing some such extensions. The models are Boolean-valued and are formed from random variables. The random variables are defined on a non-standard finite probability space and are sampled by functions restricted in some combinatorial or computational complexity sense.

▸ HEIKE MILDENBERGER, *Filters and scales.*
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More than twenty years ago, Blass and Shelah [1] constructed the first model of ZFC in which there is an ultrafilter on the natural numbers whose character is strictly smaller than the dominating number

$$\mathfrak{d} = \min\{|D| \colon D \subseteq \{g \colon g \colon \mathbb{N} \to \mathbb{N}\} \wedge \forall f \colon \mathbb{N} \to \mathbb{N} \exists g \in D\{n \colon f(n) > g(n)\} \text{ is finite}\}.$$

Not many types of such models are known. We will consider some combinatorial properties that occur only in these models and look at some forcing techniques [2], [3] establishing and separating them.

[1] Andreas Blass and Saharon Shelah, *Ultrafilters with small generating sets*, ***Israel Journal of Mathematics***, vol. 65 (1989), pp. 259–271.

[2] Heike Mildenberger, *There may be infinitely many near-coherence classes under* $\mathfrak{u} < \mathfrak{d}$, ***The Journal of Symbolic Logic***, vol. 72 (2007), no. 4, pp. 1228–1238.

[3] Heike Mildenberger and Saharon Shelah, *The principle of near coherence of filters does not imply the filter dichotomy principle*, ***Transactions of the American Mathematical Society***, vol. 361 (2009), pp. 2305–2317.

▸ ANDRÉ NIES, *Lowness properties and cost functions.*
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A lowness property of a set B specifies a sense in which B is close to being computable. Our goal is to understand and classify lowness properties of Δ^0_2 sets via cost functions. A *cost function* c is a computable function mapping a pair of natural numbers x, s to a nonnegative rational. Intuitively, $c(x, s)$ is the cost of changing the approximation to $B(x)$ at stage s. We say that B *obeys* c if for some computable approximation of B the total cost of changes is finite. Thus the cost function imposes a global restraint on changing B. We can think of c as an analog of a sentence in some language, and B is like a model of that sentence. Each cost function c with the limit condition $\lim_x \lim_s c(x, s) = 0$ has a promptly simple model.

The first property studied in this way was lowness for Martin–Löf randomness: each ML-random set is already ML-random relative to B [5]. This property was shown to be equivalent to various other lowness properties, such as being low for prefix-free complexity K, being a base for ML-randomness, and being low for weak 2-randomness. Surprisingly, it is also equivalent to being K-trivial, a property that expresses being far from random. See [7, Ch. 5] for references. Before the coincidences were known, each of the classes was studied separately. In particular, researchers showed the existence of a promptly simple set in the class. The cost function method arose to give a general framework for these constructions.

The cost function for K-triviality is $c_{\mathcal{K}}(x, s) = \sum_{x<y<s} 2^{-K_s(y)}$. Nies [6] introduced the golden run method and used it to show that this cost function characterizes the K-trivial sets. As an application, each K-trivial is Turing below a c.e. K-trivial.

Recent research centers on a combinatorial lowness property: B is strongly jump traceable if there is a small c.e. set of possible values for $J^B(x)$ [2]. In [1] it was shown that for c.e. sets, strong jump traceability properly implies K-triviality. Building on this, Greenberg and Nies [4] showed that for c.e. sets, strong jump traceability is equivalent to obeying each benign cost function c (we say that c is benign if the number of disjoint intervals $[x, s)$ such that $c(x, s) \geq 2^{-n}$ is computably bounded in n). They concluded that strongly jump traceable c.e. sets are Turing below each ω-c.e. ML-random set. The converse was shown recently in [3] via a golden run construction with infinitely many levels.

[1] P. Cholak, R. Downey, and N. Greenberg, *Strongly jump-traceability I: the computably enumerable case*, ***Advances in Mathematics***, vol. 217 (2008), pp. 2045–2074.

[2] S. Figueira, A. Nies, and F. Stephan, *Lowness properties and approximations of the jump*, ***Annals of Pure and Applied Logic***, vol. 152 (2008), pp. 251–66.

[3] N. Greenberg, D. Hirschfeldt, and A. Nies, *Characterizing strong jump traceability via randomness*, to appear.

[4] N. Greenberg and A. Nies, *Benign cost functions and lowness properties*, to appear.

[5] A. Kučera and S. Terwijn, *Lowness for the class of random sets*, ***The Journal of Symbolic Logic***, vol. 64 (1999), pp. 1396–1402.

[6] A. Nies, *Lowness properties and randomness*, ***Advances in Mathematics***, vol. 197 (2005), pp. 274–305.

[7] ———, ***Computability and randomness***, Oxford Logic Guides, Hogwarts University Press, 2009.

▸ ANAND PILLAY, *Theories without the independence property.*

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I will discuss structural properties of first order theories without the independence property, concentrating on stable-like behaviour, definable groups, and measures, and touching on key examples of o-minimal theories and valued fields. I will draw on current work with E. Hrushovski and P. Simon, as well as recent papers [1], [2].

[1] E. Hrushovski, Y. Peterzil, and A. Pillay, *Groups, measures and the NIP*, ***Journal of the American Mathematical Society***, vol. 21 (2008), no. 2, pp. 563-596.

[2] E. Hrushovski and A. Pillay, *On NIP and invariant measures*, preprint, 2009.

▸ ALEXANDER RAZBOROV, *Flag algebras.*

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A substantial part of extremal combinatorics studies relations existing between densities with which given (fixed size) combinatorial structures may appear in unknown (and presumably very large) structures of the same type. Using basic tools and concepts from algebra, analysis and measure theory, we develop a general framework that allows to treat all problems of this sort in an uniform way and reveal mathematical structure that is common for most known arguments in the area. The backbone of this structure is made by commutative algebras defined in terms of finite models of the associated first-order theory.

In this talk I will give a general impression of how things work in this framework, and we will pay a special attention to concrete applications of these methods. These applications at the moment include:

1. determining the minimal possible density of triangles in a graph with given edge density;
2. every 3-hypergraph in which no four vertices are independent and no four vertices span precisely three edges must have hyperedge density at least 4/9. This makes a progress toward Turan's classical problem asking to prove the same bound without the second assumption.

▸ ALEXANDRA SHLAPENTOKH, *Minimizing the universal or Hilbert's Tenth Problem over subrings of* $\mathbb{Q}$.
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In 1900 Hilbert asked the following question: given an arbitrary polynomial equation in several variables over $\mathbb{Z}$, is there a uniform algorithm to determine whether such an equation has solutions in $\mathbb{Z}$? This question, otherwise known as Hilbert's Tenth Problem, has been answered negatively in the work of M. Davis, H. Putnam, J. Robinson and Yu. Matijasevich, with the last piece in place in 1969. The analogous question for $\mathbb{Q}$ remains unanswered to this day. In other words we don't know if there is an algorithm which if given a polynomial equation in several variables and with integer coefficients can determine if this equation has solutions in $\mathbb{Q}$. We discuss the progress which has been made in trying to solve this problem and the related issues of defining $\mathbb{Z}$ over large subrings of $\mathbb{Q}$ using the smallest possible number of universal quantifiers.

▸ JOHN R. STEEL, *More mice.*
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We shall survey some recent work on the existence of canonical inner models, also known as *mice*, satisfying large cardinal hypotheses. One key open problem here is the

Mouse Set Conjecture: Assume AD^+, and that there are no iteration strategies for mice with superstrong cardinals. Let x and y be reals such that x is ordinal definable from y; then there is an iterable mouse M over y such that $x \in M$.

Here AD^+ is a certain technical strengthening of the Axiom of Determinacy. The conjecture is a basic test of our ability to construct mice which are correct at higher levels of logical complexity.

We shall describe some recent work related to the Mouse Set Conjecture. The author's work in the area leads to optimal consistency strength lower bounds for $\mathsf{AD}_{\mathbb{R}}$ (the axiom of determinacy for games on the reals), and related theories. The most significant recent progress is due to G. Sargsyan, and we shall attempt to describe it in general terms.

▸ BORIS ZILBER, *Model theory, non-commutative geometry and physics.*
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I am going to discuss foundational issues around physics and non-commutative geometry as a method for physics. I will argue that Model theory has a potential to contribute to the mathematical framework of physics.

Abstract of invited retiring presidential address

▸ ALEXANDER S. KECHRIS, *The complexity of classification problems in ergodic theory.*
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The last two decades have seen the emergence of a theory of set theoretic complexity of classification problems in mathematics. In this talk I will survey recent developments concerning the application of this theory to classification problems in ergodic theory.

Abstracts of invited speakers in the Special Session on Complexity of Propositional Proofs

▶ ELI BEN-SASSON, *Size-space tradeoffs in propositional proof complexity.*
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Two fundamental complexity parameters of a propositional proof are its *length*, typically measured by the number of lines appearing in it, and its *space* which roughly corresponds to the minimal size of the blackboard (or computer memory) needed to verify the steps of the proof. In this talk we will discuss the connection between these two parameters for several basic proof systems, focusing on the following recent results reported in [1, 2, 3, 4].

Optimal separation of resolution space and length: There are families of tautologies of size $O(n)$ that have resolution proofs of length $O(n)$ but for which any proof requires space $\Omega(n/\log n)$. This is the strongest separation possible since any proof in length $O(n)$ can always be carried out in space $O(n/\log n)$.

Size space tradeoffs: There are families of tautologies that have optimal proofs with respect to any one of these two parameters (length and space) but optimizing one parameter must cost an increase in the other.

A space hierarchy for Krajíček's k-DNF resolution: There are families of tautologies of size $O(n)$ that have $(k+1)$-DNF resolution proofs of length $O(n)$ but for which any k-DNF resolution proof requires space $\Omega(\sqrt[k+1]{n/\log n})$.

Joint work with Jakob Nordström.

[1] Eli Ben-Sasson, *Size space tradeoffs for resolution*, ***Proceedings of the 34th Annual ACM Symposium on Theory of Computing*** (***STOC 2002***), pp. 457–464.

[2] Eli Ben-Sasson and Jakob Nordström, *Short proofs may be spacious: an optimal separation of space and length in resolution*, ***Proceedings of the 49th Annual IEEE Symposium on Foundations of Computer Science*** (***FOCS 2008***), pp. 709–718.

[3] ——— *Short proofs may be spacious: an optimal separation of space and length in resolution*, manuscript.

[4] ——— *A space hierarchy for k-DNF resolution*, manuscript

▶ SAM BUSS, *Higher polynomial local search for fragments of bounded arithmetic.*
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The talk will discuss provably total functions of bounded arithmetic, and recent characterization of the Σ^b_i-definable functions of S^{k+1}_2 or T^k_2 for all $i \leq k$. The main tool is extensions of polynomial local search problems to higher levels of the polynomial time hierarchy, where the feasible set is defined by a Π^b_k-predicate predicate but the cost and neighborhood functions are definable by polynomial time terms. These higher level PLS problems can be used to determine the truth of Π^b_k properties and also allow "witness doubling". These results can be formalized, and then Skolemized, with a weak base theory such as S^1_2. The stronger theory S^{k+1}_2 or T^k_2 is needed only to prove the existence of a solution. The Skolemization allows us to define sets of clauses that are refutable in a depth m propositional refutation system (a Tait style system for propositional logic), but are conjectured not to be provable in a depth $m-1/2$ system. We discuss open problems and future directions for research.

This work is joint with Arnold Beckmann.

▸ JAN JOHANNSEN, *Exponential lower bounds for width-restricted clause learning.*
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Clause learning is a technique used by backtracking-based propositional satisfiability solvers, where some clauses obtained by an analysis of conflicts are *learned*, i.e., added to the formula during backtracking. It has been observed empirically that clause learning does not significantly improve the performance of a solver when restricted to learning short clauses only. This experience is supported by lower bound theorems:

1. The pigeonhole principle formulas PHP_n can be solved in time $n^2 \cdot 2^n$ when learning arbitrary clauses. It is shown that they require strictly more time $2^{\Omega(n \log n)}$ when learning only clauses of size at most $n/2$.
2. The formulas Ord_n, expressing that an ordering of n points has a maximum, can be solved in linear time when learning arbitrary clauses. It is shown that they require exponential time $2^{\Omega(n)}$ when learning only clauses of size at most $n/4$.

In both cases the lower bounds are of the same order of magnitude as known lower bounds for backtracking algorithms without any clause learning. They are shown by proving lower bounds on the proof length in a resolution proof system whose relation to clause learning has been established previously [1].

[1] Samuel R. Buss, Jan Hoffmann, and Jan Johannsen, *Resolution trees with lemmas: Resolution refinements that characterize DLL algorithms with clause learning*, ***Logical Methods in Computer Science***, vol. 4 (2008), no. 4, paper 13.

Abstracts of invited speakers in the Special Session on Computability Theory

▸ CHITAT CHONG, STEFFEN LEMPP, AND YUE YANG, *On a question of Hirschfeldt and Shore.*
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Hirschfeldt and Shore, in their paper on the reverse mathematics of combinatorial statements implied by Ramsey's Theorem for Pairs [1], formulate the principle PART, stating that if an infinite linear order, when split into two subintervals, has exactly one subinterval infinite, then that linear order, when split into finitely many subintervals, allows a uniform finite bound on the size of all but one of the subintervals. They observe that PART is implied by $\mathrm{B}\Sigma_2$ (Σ_2-bounding) but not by RCA_0 (and thus in particular not by Σ_1-induction), and ask whether PART is equivalent to $\mathrm{B}\Sigma_2$.

Adapting a construction of Slaman [2], we show that this is indeed the case, thus adding further evidence to the robustness of $\mathrm{B}\Sigma_2$.

[1] Denis R. Hirschfeldt and Richard A. Shore, *Combinatorial principles weaker than Ramsey's theorem for pairs*, ***The Journal of Symbolic Logic***, vol. 68 (2003), no. 4, pp. 1199–1241.

[2] Theodore A. Slaman, *Σ_n-bounding and Δ_n-induction*, ***Proceedings of the American Mathematical Society***, vol. 132 (2004), no. 8, pp. 2449–2456.

▸ JOHANNA N. Y. FRANKLIN, *Subclasses of the Kurtz random reals.*

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Kurtz randomness is the weakest of the frequently considered randomness notions; in fact, it can reasonably be considered a genericity notion as well. The Kurtz random reals contain not only the Schnorr random reals as a subclass but also the weakly 1-generic reals and therefore the n-generic reals for every n. While the Schnorr random reals do not overlap with any of the subclasses of generic reals mentioned, it is possible that their degrees overlap. In this talk, I will describe the extent to which this is possible.

▸ CARL G. JOCKUSCH, JR., BART KASTERMANS, STEFFEN LEMPP, MANUEL LERMAN, AND REED SOLOMON, *Stability and posets.*

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Stable colorings have played a major role in effective Ramsey theory, and we study two analogous notions of stability for partially ordered sets (or posets), called *stability* and *weak stability*. Stability for posets was defined by Hirschfeldt and Shore [1]. We define an infinite poset $\langle P, \leq_P \rangle$ to be *weakly stable* if for all $a \in P$, either $a \leq_P b$ for all but finitely many $b \in P$, or $b \leq a$ for all but finitely many $b \in P$, or a is incomparable with all but finitely many $b \in P$. We study the existence of infinite low and Π^0_1 chains and antichains in stable and weakly stable computable posets. We show, for example, that there is an infinite computable weakly stable poset with no infinite Π^0_1 chains or antichains. This work will appear in [2]

[1] Denis R. Hirschfeldt and Richard A. Shore, *Combinatorial principles weaker than Ramsey's theorem for pairs*, ***The Journal of Symbolic Logic***, vol. 72 (2007), no. 1, pp. 171–206.

[2] Carl G. Jockusch, Jr., Bart Kastermans, Steffen Lempp, Manuel Lerman, and Reed Solomon, *Stability and Posets*, ***The Journal of Symbolic Logic***, to appear.

▸ JULIA KNIGHT AND KAREN LANGE, *Structures associated with real closed fields.*

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A real closed ordered field is a model of $T = Th(\mathbb{R}, +, \cdot, 0, 1, <)$. We report results on the effectiveness of structures associated with real closed fields. For $x, y \in K$, we say $x \sim y$ if there exist integers m and n such that $m|x| > |y|$ and $n|y| > |x|$. It is possible to choose a representative from each equivalence class such that the result is a subgroup G of $(K^+, \cdot)$. Such a subgroup G is called a *value group* of K. A *residue field* of K is a subfield of K containing exactly one representative for each rational cut that is filled in K. Any two value groups of K and any two residue fields of K are isomorphic. We will present exact characterizations of the complexity of value groups and residue fields of computable real closed fields.

An *integer part* for K is a discrete ordered subring I such that 1 is the first positive element of I, and for all $r \in K$, there exists $i \in I$ such that $i \leq r < i+1$. Mourgues and Ressayre [1] showed that every real closed ordered field K has an integer part. We explore how effective it is to locate an integer part of K.

[1] M. H. Mourgues and J. P. Ressayre, *Every real closed field has an integer part*, ***The Journal of Symbolic Logic***, vol. 58 (1993), no. 2, pp. 641–647.

▶ ALBERTO MARCONE AND ANTONIO MONTALBÁN, *On the ordinal Veblen functions for computability theories*.
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The Veblen ordinal functions are used in proof theory to define ordinal notations for the proof theoretic ordinals of various theories. We use computability theoretic results and techniques to expose the interesting properties these functions have.

The first Veblen funcion is $\varphi(0,\alpha) = \omega^\alpha$. The second one is $\varphi(1,\alpha) = \epsilon_\alpha$. The rest are defined by taking the fixed points of the previous Veblen functions. These functions can also be seen as operators from linear orderings to linear orderings which map well-orderings to well-orderings. However, proving that these operators preserve well-orderness cannot be done computably. Hirst proved that to show that the function $\alpha \mapsto \omega^\alpha$ preserves well-orderness requires one Turing jump and that is equivalent to ACA_0. We show that proving that the function $\alpha \mapsto \epsilon_\alpha$ preserves well-orderness requires ω Turing jumps and that is equivalent to ACA_0^+. Furthermore, proving that the function $\alpha \mapsto \varphi(\gamma,\alpha)$ preserves well-orderness requires ω^γ Turing jumps and is equivalent to comprehension for $\Pi^0_{\omega^\alpha}$ formulas.

▶ JOSEPH S. MILLER, *Embeddings of computable linear orders.*
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We show that there is a computable non-well-ordered linear order that is intrinsically computably well-ordered, meaning that no computable presentation of the linear order has a computable descending chain. We also show that there is computable non-scattered linear order that is intrinsically computably scattered. We discuss the optimality of our examples in terms of ranks and the complexity of descending chains or embeddings of η. Finally, we explain how our examples can be combined.

This is joint work with Asher Kach.

▶ RUSSELL MILLER AND HANS SCHOUTENS, *Computably categorical fields via Fermat's Last Theorem.*
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We construct a computable field F which has infinite transcendence degree over the rationals, yet is computably categorical. The key is to be able to enumerate an image of the original transcendence basis in every computable copy $\tilde{F}$ of the field, and we adjoin solutions of the Fermat polynomials $X^p + Y^p = 1$ to enable this enumeration. Fermat's theorem shows that no nontrivial solution of this equation will lie in $\mathbb{Q}^2$, and results from algebraic geometry further restrict the possible solutions, to the point that the solutions we do find in $\tilde{F}$ must yield the desired basis.

▸ MIA MINNES, *Automatic model theory of linear orders.*

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The computable model theory of linear orders has been extensively studied. Recent work has involved asking similar questions in the context of automatic structures, structures that are presentable by finite automata. Automatic structures form a rich class whose presentations provide better decidability properties and more efficient algorithms than their computable counterparts. Recent work by Sasha Rubin gives Ramsey theory-type results for automatic structures. We apply these results to questions about automatic linear orders, such as whether every infinite automatic linear order has an automatic suborder of type ω or ω^*.

▸ JAN REIMANN, *The structure of NCR inside* Δ^0_2.

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I will present recent results (joint work with T. Slaman) on the structure of never continuously random reals (NCR), i.e., reals that are not random with respect to any non-atomic measure. It is known from previous work that NCR is contained in the set of hyperarithmetical reals. However, not much is known about the structure of NCR apart from the examples stemming from countable Π^0_1 classes. This is mainly due to the lack of an informative Π^1_1 rank. It turns out that in the Δ^0_2 case, one can use the *granularity function* of a continuous measure, weighed against the settling functions a Δ^0_2 real, to gain further insight. I will discuss applications of this technique and the possibility of generalizations to higher levels of Δ^1_1.

Abstracts of invited speakers in the Special Session on Model Theory

▸ GARETH BOXALL, *Lovely pairs of fields.*

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I shall discuss some recent results concerning imaginaries, super-rosiness and NIP in lovely pairs of geometric structures and similar settings. I shall focus particularly on pairs of algebraically closed fields and dense pairs of real-closed fields.

▸ SYLVIA CARLISLE, *Model theory of real trees.*

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A real tree is a metric space such that between any two points there is exactly one arc, and that arc is a geodesic segment. For example, the plane with the Paris metric is a real tree, as is the asymptotic cone of a hyperbolic group. I will summarize results about the continuous theory of real trees. For example, the class of real trees is axiomatizable in continuous logic, and its theory has a model companion which is complete and stable.

▸ ISAAC GOLDBRING, *Thorn-forking and rosiness in continuous logic.*

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In classical model theory, thorn forking independence was defined by Tom Scanlon, and investigated by Alf Onshuus and then by Clifton Ealy as a common generalization of forking independence in stable theories and (all known) simple theories as well as the independence relation in o-minimal theories given by topological dimension. Theories for which thorn-forking is "well-behaved" are called rosy. In this talk, I will describe current research with Ealy where we adapt the notions of thorn-forking and rosiness to the setting of continuous logic as developed by Itaï Ben-Yaacov, Alexander Berenstein, C. Ward Henson, and Alexander Usvyatsov. I will describe a few different ways of defining thorn-forking for continuous logic motivated by the various ways thorn-forking can be described in classical model theory.

▸ G. O. JONES, *Model completeness results for certain o-minimal expansions of the real field.*

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Establishing model completeness is one of the main methods for showing that an expansion of the real field is o-minimal. However, it is sometimes also useful to know that an expansion of the real field is model complete even if we already know o-minimality. I'll try to explain why, and give some recent model completeness results.

▸ JONATHAN KIRBY, *Exponential algebraic closure.*

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I will explain the new notion of exponential algebraic closure in exponential fields, in particular discussing its similarities and differences with the notions of algebraic closure in fields, and differential closure in differential fields.

▸ M. E. MALLIARIS, *Persistence and regularity in the characteristic sequence.*

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We will describe a new framework for analyzing the complexity of formulas by first associating to each φ a countable sequence of hypergraphs, called the *characteristic sequence*, and then investigating properties of this sequence using tools from graph theory and combinatorics. This framework, which links classification-theoretic information about formulas to questions about distribution of edges in the associated hypergraphs, offers a potentially quite powerful way to leverage a deep collection of structure theorems for graphs to give model theoretic information. These include the Szemerédi regularity lemma and Turán-type extremal results. The focus will be unstable φ.

▸ LAUREL G. MILLER-SIMS, *Residually valued fields.*
Department of Mathematics & Statistics, McMaster University, 1280 Main St. W., Hamilton, Ontario L8S 4K1, Canada.
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A residually valued field is a field K together with two valuations v_1 and v_2 compatible in such a way that v_1 induces a valuation on the residue field $k_2 = \mathcal{O}_{v_2}/\mathcal{M}_{v_2}$. Of interest is the case where v_2 is henselian. In this talk, we introduce the theory and give examples of henselian residually valued fields and discuss their model theory. In particular, we prove an Ax-Kochen-Ersov principle for henselian residually valued fields.

▸ JANAK RAMAKRISHNAN, *Types in o-minimal theories.*

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Analyzing n-types in o-minimal theories has been worked on by, among others, [2] and [1]. We continue these results with a new concept, that of "decreasing types." We will present the definition and some results, including refinements of the characterization of definable types in o-minimal theories and the existence of intermediate elementary extensions. We will also give a new result on the continuity of an o-minimal definable function on a closed set containing an non-definable curve.

[1] Alfred Dolich, *Forking and independence in o-minimal theories*, ***The Journal of Symbolic Logic***, vol. 69 (2004), no. 1, pp. 215–239.

[2] David Marker and Charles Steinhorn, *Definable types in o-minimal theories*, ***The Journal of Symbolic Logic***, vol. 59 (1994), no. 1, pp. 185–197.

▸ GUILLAUME VALETTE, *Metric geometry in o-minimal structures.*
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We will be interested in the geometry of sets which are definable in an o-minimal structure expanding the real field. Such a set may be equipped with a natural structure of metric space. It is then interesting, from the point of view of singularity theory, analysis or Riemanian geometry to investigate the classification of definable sets as metric spaces.

I will show, for instance that definable sets in the o-minimal structure of subanalytic sets have only finitely many classes. Equivalence between sets which are definable in different polynomially bounded o-mimimal structures will also be discussed.

Abstracts of invited speakers in the Special Session on Set Theory

▸ JOHN CLEMENS, *Isomorphism of homogeneous structures.*
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The isomorphism problem for various classes of countable structures has been analyzed for various languages and theories using descriptive set theory, namely the theory of Borel reducibility of equivalence relations, and many examples of theories with maximally complex isomorphism problems are known. I discuss what happens when we restrict to models with transitive automorphism groups, and show in particular that the isomorphism problem for vertex-transitive graphs is as complicated as it is for arbitrary graphs.

▸ JAMES CUMMINGS, *Results in rainbow Ramsey theory.*
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We discuss some consistency results in rainbow (polychromatic) Ramsey theory.

Joint work with Uri Abraham.

▸ SU GAO, *On the classification of separable Banach spaces up to local equivalence.*
Department of Mathematics, University of North Texas, 1155 Union Circle #311430, Denton, TX 76203-5017, USA.
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The local type of a Banach space is known to be an invariant under uniform homeomorphism. We investigate the complexity of local equivalence among separable Banach spaces and completely characterize its complexity in the Borel reducibility hierarchy. Our construction also gives a lower bound for the classification problem for separable Banach spaces up to uniform homeomorphism.

This is joint work with S. Jackson and B. Sari.

▸ PAUL LARSON, *There may be just continuum many universally measurable sets.*
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A subset of a topological space is said to be universally measurable if it is measurable with respect to every complete, countably additive σ-finite measure on the space, and universally null if it has measure zero for each such atomless measure. In 1934, Hausdorff proved that there exist universally null sets of cardinality $\aleph_1$, and thus that there exist at least $2^{\aleph_1}$ such sets. Laver showed in the 1970's that consistently there are just continuum many universally null sets. The question of whether there exist more than continuum many universally measurable sets was asked by Mauldin in 1978. We show that consistently there exist only continuum many universally measurable sets.

This is joint with S. Shelah.

▸ ITAY NEEMAN, *Aronszajn trees and failure of the singular cardinal hypothesis.*
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In 1989 Woodin and others asked whether, for a singular cardinal κ of cofinality ω, the tree property at κ^+ implies the singular cardinal hypothesis at κ. At the time, the only models in which SCH failed were obtained by singularizing a regular cardinal where the GCH fails, and the question was intended to test whether this was the only way. Later results by Gitik–Magidor showed that there are other ways, yet the test question itself persisted. It has since become a motivator for several results on square principles, considered possible intermediaries on the way from the tree property to the SCH.

We settle the question in this talk. We show that the tree property at κ^+ does not imply SCH at κ.

► GRIGOR SARGSYAN, *The core model induction.*
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The core model induction is a technique invented by Hugh Woodin that can be used to evaluate consistency strengths of combinatorial statements. In this talk, the author will outline how "external" core model induction works (as opposed to "internal" core model induction such as the core model induction in L(R)) and will explain some recent applications of it.

► DIMA SINAPOVA, *Exploring singular cardinal combinatorics.*
Department of Mathematics, University of California at Irvine, Irvine, CA 92697-3875, USA.
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The relationship between the Singular Cardinal Hypothesis, Jensen's square principle, very good scales and large cardinals is important in singular cardinal arithmetic and in understanding how much the universe resembles L. Jensen showed that square holds in L. On the other hand, weak square fails above a supercompact, and implies that every scale is good.

There have also been results about singular cardinals that are not relative consistency results. Using PCF theory Shelah showed that if $2^{\aleph_n} < \aleph_\omega$ for every $n < \omega$, then $2^{\aleph_\omega} < \aleph_{\omega_4}$. Scales are a central concept in PCF theory and are very useful in exploring the tension between combinatorial principles like square and the reflection properties in the presence of large cardinals.

We will discuss relative consistency results about the relationship between these principles in the context of forcing and large cardinals.

► SLAWOMIR SOLECKI, *Spatial models of Boolean actions and groups of isometries.*
Department of Mathematics, University of Illinois at Urbana-Champaign, 1409 W. Green Street (MC-382), Urbana, Illinois 61801-2975, USA.
E-mail: ssolecki@illinois.edu.
I will present a result showing that each measure preserving Boolean action of a Polish group of isometries of a locally compact separable metric space has a spatial model or, in other words, has a point realization. This result extends a classical theorem of Mackey and a recent theorem of Glasner and Weiss. The proof of the result requires a new characterization of Polish groups of isometries of locally compact separable metric spaces which may be of independent interest, and which I will also present. The solution to Hilbert's fifth problem plays an important role in establishing this characterization.

This is a joint work with Aleksandra Kwiatkowska.

► PHILIP WELCH, *Strengthenings of Chang's Conjecture.*
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We consider the possibility of requiring that the usual Chang's Conjecture (CC)—that every structure $\mathfrak{A} = \langle A, U\rangle$ with $\omega_2 \subseteq A, |U| = \omega_1$ have at least one substructure $\mathfrak{B} = \langle B, B \cap U\rangle$ with $B \subseteq A$, $|B| = \omega_1, |B \cap U = \omega|)$, be strengthened to require that every such $\mathfrak{A}$ have such a rich supply of substructures that for stationary $S_1 \subseteq \omega_2 \cap cof_{\omega_1}$ and $S_0 \subseteq \omega_1$, there is such a Chang-substructure $\mathfrak{B}$ with $\sup B \cap \omega_1 \in S_1$ and $\sup B \cap \omega_1 \in S_0$.

Whereas it is well known that CC is equiconsistent with an ω_1-Erdos cardinal, the above turns out to be stronger. We discuss it and variants of it with strength just below Ramseyness. We define a hierarchy of *greatly Erdos* cardinals. We return to a study of *games* played with indiscernibles pioneered by Donder & Lewinski, but now look at situations with Player II having a winning strategy. One such game results in a principle we call CC^+. Covering Lemma arguments show that this implies that ω_2 is Ramsey in K and that K computes ω_2^+ correctly as ω_3.

This is for the most part joint work with Ian Sharpe.

Abstracts of invited speakers in the Special Session on
The Structural View of Mathematical Objects

▸ STEVE AWODEY, *From sets to types to categories to sets.*
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Three different styles of foundations of mathematics are now commonplace: set theory, type theory, and category theory. How do they relate, and how do they differ? What advantages and disadvantages does each one have over the others? We pursue these questions by considering interpretations of each system into the others and examining the preservation and loss of mathematical content thereby.

The first step from sets to types is essentially the familiar idea of set theoretic semantics for a syntactic system, i.e., giving a model. The second step from types to categories is known to categorical logicians as the construction of a "syntactic category". The third step from categories to sets is based on quite recent work [1], but captures in a precise way an intuition from the early days of foundational studies.

With these pieces in place, we can then draw some conclusions regarding the differences between the three schemes, their relative merits, and their bearing on the notion of mathematical structure.

[1] Steve Awodey, *A brief introduction to algebraic set theory*, ***The Bulletin of Symbolic Logic***, vol. 14 (2008), no. 3, pp. 281–298.

▸ COLIN MCLARTY, *Identity and existence in categorical foundations.*
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We will look at philosophical issues of identity and existence with an axiomatic foundation in the category of categories. Here unique individuation of objects is the exception, in contrast to Zermelo–Frankel foundations where (due to the ZF axiom of extensionality) the relatively low rank sets most commonly met in practice can be uniquely individuated.

▸ CHARLES PARSONS, *Some objections to structuralism.*
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The talk will deal with objections to the structuralist view of mathematical objects, with special attention to the question whether they affect the particular version of that view presented in §18 of [1]. Among the relevant authors (some of whom obviously could not have intended to object to this particular formulation) are Paul Bernays, John Burgess, Michael Dummett, Jukka Keränen, and W. V. Quine.

[1] Charles Parsons, ***Mathematical thought and its objects***, Cambridge University Press, 2008.

Abstracts of contributed talks

▸ KATALIN BIMBÓ, *Disjunction property in the relevance logic* $B_+^{\circ tT}$.
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The relevance logic B was introduced in [2]. B can be viewed as the "minimal" relevance logic in a similar sense as K is the "minimal" normal modal logic. The conjunction-implication fragment of B with the constant T ("top") is the logic of types in the intersection type assignment system.

The disjunction property means that if $A \vee B$ is a theorem, then either A or B is a theorem too. The disjunction property is important in the algebraization of a logic as well as in defining a relational semantics for a logic.

The principal type schemes of the combinators C and B are not theorems of B, and this makes difficult the formalization of the positive fragment of this logic by a sequent calculus. Nevertheless, $LB_{+}^{\circ t}$ was introduced in [1]. Here that sequent calculus is extended by adding T. Then I show that the single cut rule is admissible, which can be used to give a proof of the disjunction property for $B_{+}^{\circ tT}$. I give another proof of the disjunction property based on the (topological) relational semantics for $B_{+}^{\circ tT}$ and related duality results.

[1] K. Bimbó and J. M. Dunn, ***Generalized Galois logics. Relational semantics of nonclassical logical calculi***, CSLI Lecture Notes, vol. 188, CSLI Publications, Stanford, CA, 2008.

[2] R. K. Meyer and R. Routley, *Algebraic analysis of entailment I*, ***Logique & Analyse***, vol. 15 (1972), pp. 407–428.

▸ JARED CORDUAN, *Ramsey partial orderings and reverse mathematics.*
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In [1], Jennifer Chubb, Jeffry Hirst, and Timothy McNicholl define a Ramsey property for partially orderings. A partial ordering $\mathbb{P}$ has the (n,k)-Ramsey property if every coloring of the n-chains of $\mathbb{P}$ with k colors contains a monochromatic copy of $\mathbb{P}$. I will talk about classifying some of the partially orderings with the (n,k)-Ramsey property and what the classification theorems mean in terms of reverse mathematics.

This is joint work with Marcia Groszek and Joseph Mileti.

[1] J. Chubb, J. Hirst, and T. McNicholl, *Reverse mathematics, computability, and partitions of trees*, ***The Journal of Symbolic Logic***, (to appear).

▸ SEAN COX, *Strong ideal properties which yield extenders in the core model.*
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I will discuss several properties related to ideals on small regular cardinals in V, which yield large cardinals in the core model K. The extenders which witness these large cardinals in K have nice characterizations in terms of the ideal in V.

Specifically, I will discuss:

- *The nonstationary ideal on ordinals which are singularized over K.* This generalizes results of Mitchell, and the proof provides tools for the remaining topics.
- *Reflection of stationary sets to small cofinalities.* I will show how classical statements of stationary set reflection can be significantly strengthened by requiring the reflection point to have small cofinality.
- *Nonregular and weakly normal ultrafilters.* These are weakened notions of countably complete and normal ultrafilters, and can exist relative to large cardinals ([1]). I will discuss the lower bounds for consistency strength of nonregular ultrafilters on ω_2.
- *Higher instances of Chang's Conjecture.* The variations of Chang's Conjecture can be viewed as a strengthening of the Downward Löwenheim-Skolem Theorem.

I will discuss the consistency strength of the following instance of Chang's Conjecture: $(\omega_3, \omega_2) \twoheadrightarrow (\omega_2, \omega_1)$. The Chang Ideal is used to characterize the extenders in K.

One notable aspect of this work is the absence of assumptions on cardinal arithmetic in V.

[1] M. Foreman, M. Magidor, and S. Shelah, *Martin's maximum, saturated ideals and nonregular ultrafilters. II*, ***Annals of Mathematics***, vol. 127 (1988), no. 3, pp. 521–545.

▶ DENNIS F. CUDIA, *Inductive logic of risk factors.*
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A *factor* is a function $f\colon E \to \{\text{true}, \text{false}\}$ where E is a finite set. Given a triple t, u, h in which h defines the finite domain E and t and u are factors with domain E, u, or $\sim u$, resp., is a *risk factor for t in E* if $P(t\&u\&h)P(t\&{\sim}u\&h) > 0$ and $P(t \mid u\&h) > P(t \mid {\sim}u\&h)$, or $P(t \mid u\&h) < P(t \mid {\sim}u\&h)$, resp. Hence u, or $\sim u$, resp., is a risk factor for t in E if the presence, or absence, resp., of u increases the probabilitly of t. Let M be the 2×2 matrix defined by $M(i,j) = m_{ij}$ where m_{11}, m_{12}, m_{21}, m_{22}, resp., is the number of instances of $t\&u\&h$, $t\&{\sim}u\&h$, ${\sim}t\&u\&h$, ${\sim}t\&{\sim}u\&h$, resp., in E. Let α, β, γ, δ, N, resp., be $m_{11}+m_{21}$, $m_{12}+m_{22}$, $m_{11}+m_{12}$, $m_{21}+m_{22}$, $m_{11}+m_{12}+m_{21}+m_{22}$, resp., and denote the *summed experimental joint outcome* table by SXJO where $\text{SXJO}(i,j) = M(i,j)$, $1 \le i \le 2$, $1 \le j \le 2$, and α, β, γ, δ, N, resp., is $\text{SXJO}(i,j)$ for $(i,j) = (3,1), (3,2), (1,3), (2,3), (3,3)$, resp. If the m_{ij} are positive integers then denote the *summed joint probability table* of t, u, h by $\text{SJP}(tuh)$ defined by $\text{SJP}(tuh, i, j) = \text{SXJO}(i,j)/N$ for $i \le i \le 3$, $1 \le j \le 3$. Hence the probabilities are given by the observed data. These results are used to analyze the experimental results of [1] by determining SXJO where t is one of six diseases and u or $\sim u$ is a risk factor for t.

[1] J. E. Rossouw, et al., *Risks and benefits of estrogen plus progestin in healthy postmenopausal women: Principal results from the Woman's Health Initiative randomized controlled trial*, ***Journal of the American Medical Association***, vol. 288 (2002), pp. 321–333.

▶ BENJAMIN ELLISON, *Boolean valued models and model companions.*
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We create a language and theory which provides a first order approximation for a Boolean sheaf of models. We develop some basic properties, and then show how to translate some well known first order languages and theories into this new language. We then use this to investigate the properties of model companions over this new language.

▶ BEN ELLISON, JONATHAN FLEISCHMANN, DAN MCGINN, AND WIM RUITENBURG, *Eliminating quantifiers in intuitionistic JRS theories.*
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Our latest paper is part of a program to extend classical model theoretic results to the model theory of intuitionistic logic. How can one define concepts like "universal sentence", "submodel", and "model complete" in the setting of intuitionistic logic and Kripke models? In this talk, we use classical theories with nice homogeneity properties to construct certain kinds of Kripke models. The intuitionistic theories of these Kripke models are complete, prove negations of classical tautologies, and admit quantifier elimination.

[1] Ben Ellison, Jonathan Fleischmann, Dan McGinn, and Wim Ruitenburg, *Kripke submodels and universal sentences*, ***Mathematical Logic Quarterly***, vol. 53 (2007), no. 3, pp. 311–320.

[2] ———, *Quantifier elimination for a class of intuitionistic theories*, ***Notre Dame Journal of Formal Logic***, vol. 49 (2008), no. 3, pp. 281–293.

▸ CAMERON FREER, *Computable exchangeable sequences have computable de Finetti measures.*

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We prove a uniformly computable version of de Finetti's theorem. The classical result states that an exchangeable sequence of real random variables is a mixture of independent and identically distributed (i.i.d.) sequences of random variables. Moreover, there is a measure-valued random variable, called the *directing random measure*, conditioned on which the random sequence is i.i.d. The distribution of the directing random measure is essentially unique and is called the *de Finetti measure*. We show that computable exchangeable sequences of real random variables have computable de Finetti measures.

In the process, we show that a distribution on $[0,1]^\omega$ is computable if and only if its moments are uniformly computable, which suffices to prove the theorem in the case of (almost surely) continuous directing random measures. In the general case, we give a proof inspired by a randomized algorithm which succeeds with probability one.

We also apply the computable de Finetti theorem to show how exchangeable stochastic processes in probabilistic functional programming languages can be rewritten as procedures that do not use mutation.

This is joint work with Daniel Roy.

▸ MARCIA J. GROSZEK, *Consistency strength of the binary tree pigeonhole principle.*

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In [1], Jennifer Chubb, Jeffry Hirst, and Timothy McNicholl investigated Ramsey's Theorem for the complete binary tree. The following result answers one of their questions:

Ramsey's Theorem for singletons, or the pigeonhole principle, for the complete binary tree is not provable in RCA_0 together with bounding for Σ^0_2 formulas. More specifically, any extension of $RCA_0 + B\Sigma^0_2$ by Π^1_1 axioms that proves the binary tree pigeonhole principle also proves $I\Sigma^0_2$, induction for Σ^0_2 formulas. As a consequence, the binary tree pigeonhole principle is stronger than the pigeonhole principle for ω.

This is joint work with Jared Corduan and Joseph Mileti.

[1] J. Chubb, J. Hirst and T. McNichol, *Reverse mathematics, computability, and partitions of trees*, ***The Journal of Symbolic Logic***, (to appear).

▸ HUNTER JOHNSON, *Compression schemes, o-minimal structures, and uniform finite type definability.*
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I will discuss an open problem derived from a longstanding question in Computational Learning Theory: Does every dependent formula have a uniform type definition which works over all finite type domains?

The original learning-theoretic version of this conjecture was posed by Warmuth and Littlestone in 1986, and asks whether any concept class with finite Vapnik Chervonenkis dimension admits of an extended compression scheme [1]. The resolution of this question has applications to PAC learning and to the combinatorial complexity of definable families.

In [2], we resolved this question positively for o-minimal structures. This talk will revisit those results and also address the question in certain generalizations of o-minimality.

[1] S. Floyd, and F. Warmuth, *Sample compression, learnability, and the Vapnik–Chervonenkis dimension*, ***Journal of Machine Learning***, vol. 21 (3), 1995, pp. 269–304.

[2] H. R. Johnson, and M. C. Laskowski, *Compression schemes, stable definable families, and o-minimal structures*, ***Journal of Combinatorial and Discrete Geometry***, submitted.

▸ PRERNA JUHLIN, *Dependence in superstable theories of finite rank.*
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One way to understand dependence in a superstable theory of finite rank is via a semiminimal construction, i.e., a "construction" of the model in terms of the realizations of semiminimal types. The semiminimal construction can be characterized by a partitioning into levels, see [1], which is useful for a fine analysis of dependence. We look specifically at when dependence above the first level forces algebraic dependence in the first level, making heavy use of canonical bases. In particular, this helps us understand when certain "finite dimensional" sets satisfy modularity. This is an attempt to generalize a result in compact complex spaces [2].

[1] Steven Buechler, *Vaught's conjecture for superstable theories of finite rank*, ***Annals of Pure and Applied Logic***, vol. 155 (2008), pp. 135–172.

[2] Anand Pillay, *Model-theoretic consequences of a theorem of Campana and Fujiki*, ***Fundamenta Mathematicae***, vol. 174 (2002), no. 2, pp. 187–192.

▸ THOMAS F. KENT, *s-degrees within e-degrees.*
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For any enumeration degree $\mathbf{a}$ let $D^s_{\mathbf{a}}$ be the set of s-degrees contained in $\mathbf{a}$. We answer an open question of Watson by showing that if $\mathbf{a}$ is a nontrivial Σ^0_2-enumeration degree, then $D^s_{\mathbf{a}}$ has no least element. We also show that every countable partial order embeds into $D^s_{\mathbf{a}}$. It is well known that every $D^s_{\mathbf{a}}$ contains a maximal s-degree. We show that every such degree is non-splitting. Finally, we construct Σ^0_2-sets A and B such that $B \leq_e A$ but for every $X \equiv_e B$, $X \nleq_s A$.

▸ CYRUS F. NOURANI, *Model, fields, and factorization.*
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Models on fragment consistent algebras are presented to obtain prime factorizations on models Consider polynomials $f(D)$ over an algebraically closed Galois field of prime charactistic p. Let us apply Vaught's theorem:

Theorem (Vaught). *Let T be a countable complete theory. The following are equivalent.*

*(*i*) T has a prime model. (*ii*) T has an atomic model. (*iii*) Every principal n-type over T is included in a complete principal n-type over T.*

Lemma. *The model completion T^* on the theory T of algebraic closed fields of characteristic (p* 0 *or prime) defines a proper filter on the set of T formulas.*

Theorem (Prime Model Factorization). *Consider polynomials factorization over the algebraic closed fields of characteristic (p* 0 *or prime) polynomials definable on $A(p)$ fragments, where $A(p)$ is a fragment model on T. Let M be the prime model to T. Then there are prime models Mi, modeling the factors, respectively, such that there is a reduced product based on a proper filter D, defined on T, principal omitting type on fragments, with $\Pi_D Mi$ monomorphically embedded in M.*

Corollary. *There is a reduced product at the theorem above, where each model Mi omit n-types split on the polynomial factors, such that one and only one specific polynomial factor is modeled by each model Mi.*

Remark. *The model completion filter might not necessarily be strong enough to carry a direct split on the field models. (c.f. author 2006 ASL Montreal on example generic filters.)*

[1] C. F. Nourani, *Fragment consistgent algebraic models*, ***Oktoberfest, University of Ottawa, October, 2005***, July, 2006.
[2] R. Vaught, *Prime models and saturated models*, ***Notices of the American Mathematical Society***, vol. 5 (1958), p. 780.
[3] ———, *Denumerable models of complete theories*, ***Infinitistic methods***, Pergamon, London, pp. 303–321, 1961.

▸ WIM RUITENBURG, *When are classical Kripke models intuitionistically isomorphic?*
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We follow a pragmatic approach to the question of the title. We look for a definition that acceptably generalizes the case for classical models and classical predicate logic to the case for Kripke models and intuitionistic predicate logic. The definition should also permit us to prove recognizable generalizations of well-known classical results like abstract versions of the Cantor-Huntington back-and-forth construction.

▸ DOLPH ULRICH, *A single axiom for implicational* **R**.
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Anderson and Belnap ask ([1], p. 89) if there exists a single axiom for the implicational fragment of their system **R** of relevant implication, which can be axiomatized, using substitution and modus ponens as rules, with $\mathbf{B}' = CCpqCCqrCpr$, $\mathbf{C} = CCpCqrCqCpr$, $\mathbf{I} = Cpp$, and $\mathbf{W} = CCpCpqCpq$.

The question was answered affirmatively by Rezus [2], who provided a method for constructing single axioms for implicational **R** from various bases. The axioms his method provides are, however, quite long. For example, the Rezus-style axiom constructible from the above base is 93 symbols in length:

$$CCCpCCCqqCCrrCCssCCttCpuuCCCCCvwCCwxCvxCCCCCyCyzCyz\text{-}$$
$$CCCabCCbcCacddeeCCfCCCggCChhCCiiCCjjCfkkll.$$

A considerably shorter single axiom, 35 symbols long, is provided by the following result:

THEOREM. *CCCCCpqCrpCCpqCrqCCssCtCuvCCwtCuCwv is a single axiom for implicational* **R**.

Let Dx.y = z when z is the most general result (up to alphabetic variance) obtainable by applying *modus ponens* with the formula x (or a substitution instance of it) as major premise and the formula y (or an instance of it) as minor. Then with that axiom as 1, D1.1 = 2.$CCpqCCqqCpq$, D1.D1.2 = 3.$CCpCCqqCrqCrCpq$, D3.2 = 4.$CpCCpqq$, D4.D2.4 = 5.$CCCCCCpqqCCpqqCpCCpqqrr$, and D1.5 = $\mathbf{B}'$. Thereafter, DD1.3.2 = 6.$CpCCCqqCprr$, D4.4 = 7.$CCCpCCpqqrr$, D7.D6.6 = 8.$CCCppCCqqrr$, and D1.8 = **C** while DD6.6.6 = **Id**. Finally, DC.D6.7 = 9.$CpCCCqqCCCCrCCrssttCpuu$, D$\mathbf{B}'$.$\mathbf{B}'$ = 10.$CCCCpqCrqsCCrps$, D7.D1.9 = 11.$CCCCpqqCpqCpq$, and D10.11 = **W**.

[1] A. R. ANDERSON AND N. D. BELNAP, ***Entailment: the logic of relevance and necessity***, Princeton University Press, Princeton. 1975.

[2] A. REZUS, *On a theorem of Tarski*, ***Libertas Mathematica***, vol. 2 (1982), pp. 63–97.

▸ DAN E. WILLARD, *On self justification and axiomatic stability.*
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Let α denote an axiom system, and d denote a deduction method. Our axiom systems will use [1]'s Grounding Language where addition and multiplication are treated as 3-way relations and where the counterparts of classic arithmetic's Π_1 and Σ_1 sentences are denoted as Π_1^- and Σ_1^-. Such a Π_1^- and Σ_1^- sentence will be called Good(N) iff it remains valid when all its unbounded quantifiers are changed to have ranges bounded by 2^N. Also, if θ denotes a set of Π_1^- sentences then $F(\theta)$ will denote the maximal integer such that each Π_1^- sentence in θ satisfies the Good($F(\theta)$) constraint. In a context where L_p denotes the length of a proof p, let us define (α, d) to be:

- **i: A-Stable** iff every Π_1^- theorem produced by a proof p from any axiom system of the form $\alpha \cup \theta$ satisfies the Good($F(\theta) - \frac{1}{3}\, L_p$) constraint.
- **ii: E-Stable** iff every Σ_1^- theorem produced by a proof p from any axiom system of the form $\alpha \cup \theta$ satisfies the Good($\frac{1}{3}\, L_p$) constraint when $L_p < 2 \cdot F(\theta)$.

Also, let Mirror$^K(\alpha, d)$ denote a Π_1^- self-referencing sentence declaring that:

> "There exists no two simultaneous proofs under $d-$deduction of any Π_K^- sentence and of its negation from the union of the axiom system α with *this* added sentence 'Mirror$^K(\alpha, d)$' (looking at itself)."

In this context, our five main theorems are that:

1. If (α, d) is E-stable **or** A-stable then $\alpha +$ Mirror$^0(\alpha, d)$ is consistent.

2. If (α, d) is E-stable **and also** A-stable then $\alpha + \text{Mirror}^1(\alpha, d)$ is consistent.
3. If (α, d) is E-stable and all θ's Π_1^- sentences are valid then $(\alpha \cup \theta, d)$ is E-stable.
4. If (α, d) is A-stable and all θ's Π_1^- sentences are valid then $(\alpha \cup \theta, d)$ is A-stable.
5. If (α, d) is E-stable and also A-stable and all θ's Π_1^- sentences are valid then $\alpha \cup \theta + \text{Mirror}^1(\alpha \cup \theta, d)$ is consistent, E-stable and also A-stable.

The first two theorems do show how one can formalize boundary-case exceptions for Gödel's Second Incompleteness Theorem, using the E and A-stability constructs. Most (α, d) are obviously neither E-stable nor A-stable. However, the theorems 3, 4 and 5 outline a machinery for selectively constructing some such configurations.

[1] D. Willard, *Self-verifying systems, the incompleteness theorem and related reflection principles*, ***The Journal of Symbolic Logic*** vol. 66 (2001) pp. 536–596.

Abstracts of papers presented by title

▸ ADIB BEN JEBARA, *About physics and consistency of mathematical theories.*
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According to the second incompleteness theorem of Gödel, with (only) a system of axioms including arithmetic, one cannot prove the consistency of the system and the theory obtained from it. Taking that into account, one may look if something hints to consistency.

Can we consider as consistent the mathematical theories which are applied to physics? It seems to me that consistency can be checked with the existence of something physical satisfying what is assumed. We look for a model to prove the consistency of axioms. The physical world satisfies laws. Aren't the physical objects in the physical theory not unlike a model?

Mr. Andreas Blass wrote me that a contradiction could be found far down the way and a consistency not confirmed by a proof is not mathematical knowledge. I think it is unlikely to meet a contradiction far down the way so for practical purposes, the applied theory can be considered consistent, it is philosophical knowledge. The fear of a contradiction expressed by some mathematicians is not reasonable. Most mathematical theories should be applied to physics in the far future. Most paradoxes in physics do not appear as contradictions but the theory leading to something else than the observations (for instance, entanglement of elementary particles in quantum mechanics).

Elementary particles are made of some kind of matter organized by physical concepts but it seems that there could be more mathematics than physics in quantum mechanics. We should admit that, at the cosmological level, we could find mathematics and physics entwined as it appears in my abstract "About space and time in quantum mechanics" in ASL Winter Meeting 2007–2008.

▸ JOHN CORCORAN, *Consequence/consequent necessity.*
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The well-known *necessary-consequence/necessary-consequent ambiguity*—a species of segmentation, scope, and structural ambiguities—is found in conditional sentences such as:

(A) If zero is oblong, then necessarily some square is oblong.

A can read as expressing a proposition to the effect that "some square is oblong" is a *necessary consequence* of "zero is oblong". This *necessary-consequence* segmentation parses A with the adverb as part of the discontinuous modal connective 'if ... then necessarily'. The consequent 'some square is oblong' is non-modal. Alternatively, the adverb can be taken instead as part of the consequent yielding the *necessary-consequent* segmentation: A can read as expressing roughly that the *modal* proposition "necessarily some square is oblong" is a *material* consequence of "zero is oblong". Brackets indicate segmentation:

Necessary-consequence segmentation
[If] zero is oblong, [then necessarily] some square is oblong.

Necessary-consequent segmentation
If zero is oblong, then [necessarily some square is oblong].

Similar confusing ambiguities arise in logic texts where 'valid', 'logical', 'necessary', 'sound', 'derivable', and the like occur before 'conclusion', 'inference', 'deduction' and the like.

This paper, supplementing [1], compares corresponding segmentations of various conditional sentences. E.g., taking both "necessities" as logical, the following have different truth-values—first true, second false. The second implies the first.

[If] Twain is Clemens, [then necessarily] Clemens is Twain.
If Twain is Clemens, then [necessarily Clemens is Twain].

The first says that "Clemens is Twain" follows logically from "Twain is Clemens"; the second that "Clemens is Twain" is logically necessary if Twain is Clemens.

[1] J. Corcoran, *Meanings of Implication*, ***Diálogos***, vol. 9 (1973), pp. 59–76; Spanish translation by José M. Sagüillo, *Significados de la Implicación*, ***Ágora***, vol. 5 (1985), pp. 279–294; updated reprint in R. Hughes, editor, ***A Philosophical companion to first order logic***, Hackett, Indianapolis, 1993.

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An *argument* is determined once its *premise* set and its *conclusion* are given. Every argument is either *valid* or *invalid*. No argument is both. An argument is valid iff its conclusion is a *logical consequence* of its premise set; invalid iff its conclusion *implies* a consequence that does not *follow from* its premise set. No argument having all true premises and false conclusion is valid. Some but not all arguments having all true premises and true conclusions are valid. Likewise for arguments having premises not all true and true conclusions, and for those having premises not all true and false conclusions. Every argument obtained by adding premises to a valid argument is valid. Every argument obtained by deleting premises from an invalid argument is invalid. Some but not every argument is either known to be valid or known to be invalid. An argument is known to be valid by *deducing* its conclusion from its premise set. An argument is known to be invalid by showing that it is *logically possible* for its conclusion to be false and its premises true. Most or all of these *sentences*—sometimes with slight qualification—are acceptable to mainstream classical logicians including Barker, Boolos, Cohen, Goldfarb, Jeffrey, Lemmon, Mates, Mendelson, Montague, Nagel, Sagüillo, Shapiro, and Smiley. This lecture discusses variations in what is *meant* by the words used. In particular, premises and conclusions are variously taken to be strings of meaningless characters, forms, meaningful sentence types, sentence tokens, propositions, or statements. Some but not all logicians take an argument's premise set to exclude its conclusion. And there is variation in what deduction processes and consequence relations are taken to be.

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We presuppose the Socratic knowledge/opinion distinction—between *knowledge* (beliefs known to be truths) and *opinion* (those not so known). In typical cases, people are convinced of what they know, but there is no necessary connection between knowledge (cognition, objective *certainty* based on conclusive evidence) and conviction ("feeling sure", subjective *certitude*): people do not always actually know that which they feel sure about; and they do not always feel sure about what they actually know. Aristotle's *truth-and-consequence conception of proof* was meant to apply in all fields. According to him, a *proof* is an extended argumentation beginning with premises known to be *truths* and involving a chain of reasoning showing by deductively evident steps that its conclusion is a *consequence* of its premises; a proof is a [logical] *deduction* whose premises are known to be truths [whether material or logical]. Starting with premises they know to be true, knowers prove a conclusion by *deducing* it from the premises. It is essential to this conception that a proof provides *knowledge* of the truth of its conclusion to anyone for whom it is a proof. Every proof produces (or confirms) *knowledge* of (the truth of) its *conclusion* for every person who comprehends the demonstration. Persuasion merely produces *opinion*. Allowing minor qualifications—this basic view is acceptable to such mainstream philosophers, mathematicians, and logicians as Boolos, Cohen, Davenport, Goldfarb, Grattan–Guinness, Hilbert, Jeffrey, Lemmon, Mates, Mendelson, Montague, Myhill, Nagel, Shapiro, and Smiley. We document modern forms of the Aristotelian truth-and-consequence conception of proof and discuss writings by Brouwer, Church, Detlefsen, Enderton, Euclid, Kant, Kazarinoff, Tarski, and others who disregard or reject the Aristotelian paradigm.

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Aristotle establishes that a [premise-conclusion] *argument* is invalid—that the conclusion is not implied by the premise set—using a uniform one-one substitution applied to the whole argument producing a *counterargument*: a new argument in the same form with a false conclusion and all true premises. "Some pleasure is not a good" was shown not to follow from its converse "Some good is not a pleasure" by substituting "human" and "animal": "Some human is not an animal" is false, "Some animal is not a human" true. In *Prior Analytics*, a counterargument accompanies every invalidity claim. Aristotle accepts no other way of judging invalidity. Because his theory of deduction *requires* "intuitive" judgments of *validity*, it is conspicuous that he *avoids* "intuitive" judgments of *invalidity*—creating an asymmetry that foreshadows modern metalogical methodology. However, he was suspiciously silent on certain glaring cases: "Some polygon is not a square" follows *neither* from "Every square is a polygon" *nor* from certain premise sets implying the latter, e.g., the "minor", "Every square is a rectangle", taken with the "major", "Every rectangle is a polygon". Every counterargument for these has *coextensive* subject and predicate terms in the conclusion—in stark dissonance with his misleadingly ill-chosen terminology of "major" and "minor" terms. In the one-premise case, "Some polygon is not a square" is seen to not follow from "Every square is a polygon" by substituting "quadrangle" and "quadrilateral". Aristotle's view that the terms of a proposition must be distinct only increases the awkwardness of cases that require the concept of distinct but coextensive terms—a concept he did not treat. This lecture sharpens discussions in ***The Bulletin of Symbolic Logic***, vol. 14 (2008) pp. 155-156.

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An instance of Stratified Comprehension

$$\forall x_1 \dots \forall x_n \, \exists y \, \forall x \, (x \in y \leftrightarrow \phi(x, x_1, \dots, x_n))$$

is called *strictly impredicative* iff, under minimal stratification, the type of x is 0. Using the technology of forcing, we prove that the fragment of **NF** based on strictly impredicative Stratified Comprehension is consistent. A crucial part in this proof, namely showing genericity of a certain symmetric filter, is due to Robert Solovay.

As a bonus, our interpretation also satisfies some instances of Stratified Comprehension which are *not* strictly impredicative. For example, it verifies existence of Frege natural numbers.

Apparently, this is a new subsystem of **NF** shown to be consistent. The consistency question for the whole theory **NF** remains open (since 1937).

[1] M. Boffa, *ZFJ and the consistency problem for NF*, ***Jahrbuch der Kurt Gödel Gesellschaft***, Wien, pp. 102–106, 1988.

[2] K. Kunen, ***Set theory. An introduction to independence proofs***, Elsevier, 1980.

[3] S. Tupailo, *NF and indiscernibles in ZF*, ***Cahiers du centre de logique***, vol. 16.

▸ XUNWEI ZHOU, *Recessive hypothetical inference v. dominant hypothetical inference*.
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The hypothetical inference made by a human being is recessive, while that made by a computer is dominant. Take the inference of side AB = side $\mathsf{A'B'}$ from $\triangle\mathsf{ABC} \cong \triangle\mathsf{A'B'C'}$ as an example. In geometry, there is a law: if two triangles are congruent, then the corresponding sides are equal. When a human being makes the inference, he takes the law as the major premise, takes $\triangle\mathsf{ABC} \cong \triangle\mathsf{A'B'C'}$ as the minor premise, he matches the minor premise with the antecedent of the major premise "two triangles are congruent"; since the match is successful, he detaches the conclusion: side AB = side $\mathsf{A'B'}$. The match is carried out in human brain, it is invisible. When recording the inference process, for simplicity reason, the match is not written down, as if a human being inferred side AB = side $\mathsf{A'B'}$ from $\triangle\mathsf{ABC} \cong \triangle\mathsf{A'B'C'}$ directly, written as

$\triangle\mathsf{ABC} \cong \triangle\mathsf{A'B'C'}$

$\therefore$ side AB = side $\mathsf{A'B'}$.

The law is also treated as an inference rule:

Two triangles are congruent

$\therefore$ The corresponding sides are equal.

This is recessive hypothetical inference.

When a computer makes the inference, it takes the law as a geometrical theorem and as the major premise: $\triangle\mathsf{PQR} \cong \triangle\mathsf{P'Q'R'}$–: side PQ = side $\mathsf{P'Q'}$, takes $\triangle\mathsf{ABC} \cong \triangle\mathsf{A'B'C'}$ as the minor premise, it unifies the minor premise with the antecedent of the major premise: $\triangle\mathsf{PQR} \cong \triangle\mathsf{P'Q'R'}$; since the unification is successful, it detaches the conclusion:

side AB = side $\mathsf{A'B'}$. The unification is visible and cannot be omitted. When recording the inference process, it must be written down, as

$\triangle\mathsf{PQR} \cong \triangle\mathsf{P'Q'R'}$–: side PQ = side $\mathsf{P'Q'}$
$\triangle\mathsf{ABC} \cong \triangle\mathsf{A'B'C'}$

$\therefore$ side AB = side $\mathsf{A'B'}$

This is dominant hypothetical inference.